# Do you agree with her statement?

విజేత

For Feedback...
vijetha.nt@gmail.com

**B. Laxminarayana**
Maths Senior Faculty
Hyderabad
9849386253

## 1. REAL NUMBERS

### 1 MARK QUESTIONS

**1. State the Euclid's division algorithm.**
Given positive integers $a$ and $b$, there exist unique pair of integers $q$ and $r$ satisfying $a = bq + r$, $0 \le r < b$ (Where $a$ = Dividend, $b$ = Divisor, $q$ = Quotient, $r$ = remainder)

**2 State the Fundamental Theorem of Arithmetic.**
Every composite number can be expressed (factorized) as a product of its primes, and this factorization is unique, apart from the order in which the prime factors occur.

**3. Varshitha said that sum of two irrational numbers need not be a irrational number. Give an example**
**Sol:** Yes! Some cases it is true,
for example Let $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are two irrational numbers, then the sum of them is $2 + \sqrt{3} + 2 - \sqrt{3} = 4$ is a rational number.

**4. Trishika said that 30, 030 is a product of consecutive prime numbers. Do you agree with her statement explain?**
**Sol:** Yes! 30, 030 can be expressed as a product of consecutive prime numbers as
$30,030 = 2 \times 3 \times 5 \times 7 \times 11 \times 13$

**5. Find the HCF and LCM of least prime number and least composite number.**
**Sol:** Least prime number is 2, least composite number is 4,
the HCF of 2, 4 is 2 and LCM is 4.

**6. Find the value of $\log_{5}^{625} - \log_{5}^{125}$?**
**Sol:** Given $\log_{5}^{625} - \log_{5}^{125}$
$= \log_{5}^{5^4} - \log_{5}^{5^2} = 4\log_{5}^{5} - 2\log_{5}^{5}$
$= 4 - 2 = 2$ ($\because \log_{a}^{a} = 1$)

**7. Evaluate $\frac{1}{2}\log 25 - 2\log 3 + \log 18$?**
**Sol:** Given $\frac{1}{2}\log 25 - 2\log 3 + \log 18$
$= \log 25^{\frac{1}{2}} - \log 3^2 + \log 18$
$= \log \frac{5 \times 18}{9} = \log 10 = 1$

**8. If two integers can be written as $a = x^3y^2$ and $b = xy^4$ where x, y are prime numbers. Find the HCF of (a, b).**
**Sol:** Given $a = x^3y^2$, $b = xy^4$
HCF of (a, b) = HCF of $x^3y^2, xy^4 = xy^2$

**9. If the least prime factor of a is 3 and least prime factor of b is 7 then least prime factor of (a + b) = ?**
**Sol:** Given least prime factor of a is 3 and least prime factor of b is 7 then least prime factor of (a + b) = ( 3 + 7) = 10 is 2

**10. Simplest form of $\log P + \log T = 2\log 10 + \log I - \log R$ is**
**Sol:** $\log P + \log T = 2\log 10 + \log I - \log R$
=> $\log P + \log T + \log R = \log 10^2 + \log I$
=> $\log P + \log T + \log R = \log 100 + \log I$
=> $\log PTR = \log 100I$ => $PTR = 100I$
=> $I = \frac{PTR}{100}$

### 2 MARK QUESTIONS

**1. Write the rationalizing factor of $4\sqrt{2} + \sqrt{3}$ and find their product, which is rational or irrational? Comment?**
**Sol:** Rationalizing factor of $4\sqrt{2} + \sqrt{3}$ is $4\sqrt{2} - \sqrt{3}$ now their product
$(4\sqrt{2} + \sqrt{3})(4\sqrt{2} - \sqrt{3}) = (4\sqrt{2})^2 - (\sqrt{3})^2$
$(4\sqrt{2})^2 - (\sqrt{3})^2 = 32 - 3 = 29$ is rational

**2. If $x = \log_{6}^{x} + 2\log_{36}^{x} + 3\log_{216}^{x} = 9$ then x =?**
**Sol:** $\log_{6}^{x} + 2\log_{36}^{x} + 3\log_{216}^{x} = 9$
=> $\log_{6}^{x} + \log_{6}^{x} + \log_{6}^{x} = 9$
=> $3\log_{6}^{x} = 9$ => $x = 6^3$ => $x = 216$

**3. Is it is possible to have two numbers their HCF as 16 and LCM as 380?**
**Sol:** factors of $380 = 2 \times 2 \times 5 \times 19$ the product of factors cannot consists of $19 \times 20$ so the HCF as 16 and LCM as 380 is cannot be possible for any two numbers

**4. How can you say that $\log_{2\sqrt{2}}^{512}$ is rational number?**
**Sol:** Let $\log_{2\sqrt{2}}^{512} = x$ => $(2\sqrt{2})^x = 512$
=> $(2^{\frac{3}{2}})^x = 2^9$ ($\because 512 = 2^9$)
=> $2^{\frac{3x}{2}} = 2^9$ => $\frac{3x}{2} = 9$
=> $3x = 18$ => $x = \frac{18}{3} = 6$(integer)
$\therefore \log_{2\sqrt{x}}^{512} = 6$ is a rational number.

**5. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then the relation between x, y, z is?**
**Sol:** Let If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$, then
$\log x = k(y - z)$, $\log y = k(z - x)$,
$\log z = k(x - y)$
$\log x + \log y + \log z = k(y - z + z - x + x - y)$
=> $\log xyz = k(0)$ => $\log xyz = 0$.

FOR ALL COMPETITIVE EXAMS

**6. Show that $12^n$ cannot end with the digit o or 5 for any natural number 'n'.**
**Sol:** When we express 12 as product of primes we have $12 = 2^2 \times 3$ so now $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ prime factorization of $12^n$ are 2 and 3 but not 5 so $12^n$ cannot end with digit 0 or 5.

**7. Prove that $5 + 3\sqrt{2}$ is irrational.**
**Sol:** Let us assume $5 + 3\sqrt{2}$ is rational
then $5 + 3\sqrt{2} = \frac{p}{q}$ where $q \neq 0$ and
$p, q \in z$ => $3\sqrt{2} = \frac{p}{q} - 5$
=> $3\sqrt{2} = \frac{p-5q}{q}$ => $\sqrt{2} = \frac{p-5q}{3q}$ where
$\frac{p-5q}{3q}$ is rational and $p, q \in z$,
but $\sqrt{2}$ is irrational, an irrational number never be equal rational number, so our assumption is wrong
$\therefore 5 + 3\sqrt{2}$ is irrational

**8. Show that any positive odd integer is of the form 4q +1 or 4q + 3 where q is a positive integer.**
**Sol:** Let a be a positive integer and b = 4 and $q \ge 0$ then by division algorithm
$a = 4q + r$, $0 \le r < 5$, i,e., r = 0, 1, 2, 3
if r =0 then a = 4q (even)
r =1 then a = 4q +1 (odd)
r =2 then a = 4q + 2 (even)
r =3 then a = 4q + 3 (odd)
so any positive odd integer is of the form 4q +1 or 4q + 3

**9. If $x^2 + y^2 = z^2$ then prove that $\log_{y}^{(z+x)} + \log_{y}^{(z-x)} = 2$**
**Sol:** Given $x^2 + y^2 = z^2$
=> $z^2 - x^2 = y^2$ => $(z - x)(z + x) = y^2$
take log on both sides
=> $\log[(z - x)(z + x)] = \log y^2$
=> $\log(z - x) + \log(z + x) = 2\log y$
=> $\log_{y}^{(z+x)} + \log_{y}^{(z-x)} = 2$

**10. How will you show that $(17 \times 11 \times 2) + (17 \times 11 \times 15)$ is a composite number is explain?**
**Sol:** Given $(17 \times 11 \times 2) + (17 \times 11 \times 15) = 17 \times 11(2 + 15) = 17 \times 11 \times 17$
$= 17^2 \times 11$
$\therefore$ The given number $(17 \times 11 \times 2) + (17 \times 11 \times 15)$ has more than 2 factors hence it is a composite number.

### 4 MARK QUESTIONS

**1. Use Euclid's division lemma to show that the square of any positive integer is of the form 3n or 3n + 1 for some integer n**
**Sol:** Let $a$ be any positive integer and $b = 3$, $a = 3q + r$, where $q>0$ and $0 \le r < 3$
$\therefore a = 3q$ or $3q+1$ or $3q+2$
We have the three cases.
**Case1:** when $a = 3q$ => $a^2 = (3q)^2 = 9q^2$
=> $3(3q^2) = 3n$ where $n = 3q^2$

**Case 2:** when $a = 3q+1$ => $a^2 = (3q + 1)^2$
=> $a^2 = 9q^2 + 6q + 1$
$= 3(3q^2 + 2q) + 1$
$= 3n+1$, where $n = (3q^2 + 2q)$

**Case 3:** When $a = 3q+2$
=> $a^2 = (3q + 2)^2$
$= 9q^2 + 12q + 4$
$= 3(3q^2 + 4q + 1) + 1$
$= 3n + 1$, where $m = (3q^2 + 4q + 1)$
$\therefore$ The square of any positive integer is of the form $3n$, $3n+1$ for some integer n.

**2. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$, $9m + 8$ for some integer m.**
**Sol:** Let $a$ be any positive integer and $b = 3$, $a = 3q+r$, where $q>0$ and $0 \le r < 3$
$\therefore a = 3q$ or $3q+1$ or $3q+2$
We have the three cases.
**Case1:** when $a = 3q$ => $a^3 = (3q)^3 = 27q^3$
=> $9(3q^3) = 9m$ where $m = 3q^3$

**Case 2:** when $a = 3q+1$ => $a^3 = (3q + 1)^3$
=> $a^3 = 27q^3 + 27q^2 + 9q + 1$
$= 9(3q^3 + 3q^2 + q) + 1$
$= 9m+1$, where $m = (3q^3 + 3q^2 + q)$

**Case 3:** When $a = 3q+2$
=> $a^3 = (3q + 2)^3$
$= 27q^3 + 54q^2 + 36q + 8$
$= 9(3q^3 + 6q^2 + 4q) + 8$
$= 9m+8$, where $m = (3q^3 + 6q^2 + 4q)$
$\therefore$ The cube of any positive integer is of the form $9m$, $9m+1$, $9m+8$.

**3. Prove that $(\sqrt{3} + \sqrt{5})$ is an irrational number.**
**Sol:** Let us assume $\sqrt{3} + \sqrt{5}$ is rational then
$\sqrt{3} + \sqrt{5} = \frac{p}{q}$ where $q \neq 0$ and
$p, q \in z$ => $\sqrt{3} = \frac{p}{q} - \sqrt{5}$
=> $(\sqrt{3})^2 = (\frac{p}{q} - \sqrt{5})^2$
=> $3 = (\frac{p}{q})^2 - 2\frac{p}{q}\sqrt{5} + (\sqrt{5})^2$
=> $3 = \frac{p^2}{q^2} - \frac{2p}{q}\sqrt{5} + 5$
=> $\frac{2p}{q}\sqrt{5} = \frac{p^2}{q^2} + 5 - 3$
=> $\frac{2p}{q}\sqrt{5} = \frac{p^2}{q^2} + 2$
=> $\frac{2p}{q}\sqrt{5} = \frac{p^2+2q^2}{q^2} = \frac{p^2+2q^2}{q^2} \times \frac{q}{2p}$
=> $\sqrt{5} = \frac{p^2+2q^2}{2pq}$
=> $\frac{p^2+2q^2}{2pq}$ is rational and $p, q \in z$,
but $\sqrt{5}$ is irrational,
so an irrational number never be equal rational number.
hence our assumption is wrong
$\therefore \sqrt{3} + \sqrt{5}$ is irrational

**4. If $x^4 + y^4 = 83x^2y^2$ then prove that $\log\left(\frac{x^2-y^2}{9}\right) = \log x + \log y$**
**Sol:** Given $x^4 + y^4 = 83x^2y^2$
subtract '$2x^2y^2$ on both sides
$x^4 + y^4 - 2x^2y^2 = 83x^2y^2 - 2x^2y^2$
=> $(x^2 - y^2)^2 = 9^2x^2y^2$
=> $(x^2 - y^2)^2 = 9^2x^2y^2$
=> $(\frac{x^2-y^2}{9})^2 = x^2y^2$
=> $(\frac{x^2-y^2}{9})^2 = (xy)^2$ => $\frac{x^2-y^2}{9} = xy$
=> $\log\left(\frac{x^2-y^2}{9}\right) = \log(xy)$
=> $\log\left(\frac{x^2-y^2}{9}\right) = \log x + \log y$

**5. If the area of the rectangular park is 2028 with highest possible common factor in its length and breadth be 13. Write all possible pair of dimensions.**
**Sol:** Given $l \times b = 2028$.
Where $l = 13a$, $b = 13b$,
=> $13a \times 13b = 2028$
=> $169ab = 2028$ => $ab = 12$. The suitable pairs are (1, 12) (3, 4) so required pairs are $(13 \times 1, 13 \times 12)$, $(13 \times 3, 13 \times 4)$.
(13, 56) (39, 52)

**6. Find the value of $\frac{1}{1+\log_{a}^{bc}} + \frac{1}{1+\log_{b}^{ca}} + \frac{1}{1+\log_{c}^{ab}}$?**
**Sol:** Consider $\frac{1}{1+\log_{a}^{bc}} = \frac{1}{\log_{a}^{a}+\log_{b}^{ca}}$
$= \frac{1}{\log_{a}^{abc}} = \log_{abc}^{a}$ similarly
$\frac{1}{1+\log_{b}^{ca}} = \log_{abc}^{b}$ and
$\frac{1}{1+\log_{c}^{ab}} = \log_{abc}^{c}$ then
$\frac{1}{1+\log_{a}^{bc}} + \frac{1}{1+\log_{b}^{ca}} + \frac{1}{1+\log_{c}^{ab}} =$
$\log_{abc}^{a} + \log_{abc}^{b} + \log_{abc}^{c} = \log_{abc}^{abc} = 1$